AP Rev. #9 Key

1) 

a) $a(t) = 2t\cos t^2$

$a(3) = 6\cos 9$

$\boxed{a(3) \approx -5.467}$

2) a) $W'(v) = -3.536v^{-0.84}$

$\boxed{W'(20) \approx -0.286}$ When wind velocity is 20 mph, the wind chill temperature a person feels is decreasing by 0.286°F per mph of wind velocity.

b) $\text{Avg} = \dfrac{W(60) - W(5)}{60 - 5}$

$\boxed{\approx -0.254}$

$-3.536v^{-0.84} = \dfrac{W(60) - W(5)}{60 - 5}$

$\boxed{v \approx 23.01}$

3) a) Max where $f' = 0$ and goes from + to −

@ $\boxed{x = -3, 4}$

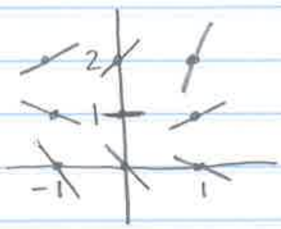
b) PI where f'' changes sign which is where f' goes from incr to decr. or decr. to incr.

PI's @ $\boxed{x = -4, -1, 2}$

c) Concave up where f' is increasing.

Positive slopes of f where f' is positive.

$\boxed{(-5, -4) \cup (1, 2)}$

4) a) 

b) $\dfrac{d^2y}{dx^2} = \dfrac{1}{2} + \dfrac{dy}{dx}$

$\boxed{\dfrac{d^2y}{dx^2} = \dfrac{1}{2}x + y - \dfrac{1}{2}}$

concave up when $\dfrac{d^2y}{dx^2} > 0$

$\rightarrow \frac{1}{2}x + y > \frac{1}{2}$

OR

$x + 2y > 1$

c) @ (0,1)

$f' = 0$

f': ⊖ 0 ⊕

Relative min.

5) a) MVT → $f'(c) = \dfrac{f(5) - f(2)}{5 - 2}$

$-1 = \dfrac{2 - 5}{5 - 2}$

$-1 = -1$ ✓

5b) $g(x) = f(f(x))$ chain rule.

$g'(x) = f'(f(x)) \cdot f'(x)$

$g'(2) = f'(f(2)) \cdot f'(2)$

$\boxed{g'(2) = f'(5) \cdot f'(2)}$

$g'(5) = f'(f(5)) \cdot f'(5)$

$\boxed{g'(5) = f'(2) \cdot f'(5)}$

Same

* Rolle's Theorem! Differentiable and $g'(2) = g'(5)$, so on $2 < k < 5$ there must be a k where $g''(k) = 0$

5c) $g'(x) = f'(f(x)) \cdot f'(x)$

$g''(x) = f'(f(x)) \cdot f''(x) + f''(f(x))\, f'(x) \cdot f'(x)$

If $f''(x) = 0 \rightarrow g''(x) = f'(f(x)) \cdot 0 + 0\left[f'(x)\right]^2$

$g''(x) = 0$

$\therefore$ g will have no changes in concavity.

6a) $h(1) = f(g(1)) - 6$
$= f(2) - 6$
$= 9 - 6$
$= 3$

$h(3) = f(g(3)) - 6$
$= f(4) - 6$
$= -1 - 6$
$= -7$

Intermediate Value Theorem:

Since $h(3) < h(r) < h(1)$, there must be: $1 < r < 3$

$-7 < -5 < 3$

6b) Mean Value Th.

$h'(c) = \dfrac{h(3) - h(1)}{3 - 1}$

$-5 = \dfrac{-7 - 3}{2}$

$-5 = -5$ ✓